International Journal of Mathematics and Physical Sciences Research ISSN 2348-5736 (Online)
Vol. 3, Issue 1, pp: (125-133), Month: April 2015 - September 2015, Available at: www.researchpublish.com

Stability Analysis From Third Order Nonlinear Evolution Equations For Counter Propagating Capillary Gravity Wave Packets In The Presence Of Wind Flowing Over Water

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Abstract: **Asymptotically exact and nonlocal third order nonlinear evolution equations are derived for two counter propagating surface capillary gravity wave packets in deep water in the presence of wind flowing over water. From these evolution equations stability analysis is made for a uniform standing surface capillary gravity wave trains for longitudinal perturbation. Instability condition is obtained and graphs are plotted for maximum growth rate of instability and for wave number at marginal stability against wave steepness for some different values of dimensionless wind velocity.**

Keywords: **Nonlinear evolution equation, capillary gravity, waves, stability analysis.**

1. INTRODUCTION

One approach to studying the stability of finite amplitude surface waves in deep water is through the application of the lowest order nonlinear evolution equation, which is the nonlinear Schr¨odinger equation.Zakharov's [10] study is along this line, allowing for finite amplitude wave trains to be subjected to modulational perturbations in two horizontal directions both along and perpendicular to the direction of the wave train. Benney and Newell [1] and Hasimoto and Onto [8] derived a single equation describing long-time evolution of the envelope of one dimensional surface-gravity wave packet on the surface of water of finite depth. Devey and Stewartson [3] extended this for a two dimensional wave packet and showed that the nonlinear evolution equation in this case is governed by two coupled equations. These equations including the effect of capillarity were derived by Djordjevic and Redekopp [5] which give the nonlinear evolution equation of a two dimensional capillary gravity wave packet. The corresponding equation for a one dimensional wave packet was obtained by Kawahara [7].

The third order nonlinear evolution equations have been derived by Pierce and Knobloch [9] for two counterpropagating capillary gravity wave packets on the surface of water of finite depth. The resulting equations are asymptotically exact and nonlocal and generalize the equations derived by Djordjevic and Redekopp [5] for counterpropagating waves. In the present paper third order nonlinear evolution equations are derived for two counterpropagating capillary gravity wave packets in the surface water of infinite depth in the presence of wind flowing over water. So this paper is an extension of the evolution equations derived by Pierce and Knobloch [9] for an infinite depth water and in the presence of wind flowing over water. These evolution equations remain valid when the dimensionless wind velocity is less than a critical velocity. This critical velocity is defined by the fact that a wave becomes linearly unstable if the wind velocity exceeds this critical velocity. From these evolution equations stability analysis is investigated for a uniform standing surface capillary gravity wave trains with respect to longitudinal perturbation. The expressions for the maximum growth rate of instability and the wave number at marginal stability are derived. Graphs are plotted for maximum growth rate of instability and for wave number at marginal stability against wave steepness for some different values of dimensionless wind velocity. It is observed that in the third order analysis the maximum growth rate of instability increases steadily with the increase of wave steepness. The growth rate is found to be appreciably much higher for dimensionless wind velocity approaching its critical value. The wave number at marginal stability has also been plotted against wave steepness for some different values of dimensionless wind velocity.

II. BASIC EQUATIONS

The common horizontal interface between water and air in the undisturbed state as z=0 plane. In the undisturbed state air flows over water with a velocity u in a direction that is taken as the x- axis. We take $z=\varsigma(x,y,t)$ as the equation of the common interface is at any time t in the perturbed state. We introduce the dimensional quantities $\tilde{\phi},\tilde{\phi}',\tilde{\varsigma},(\tilde{x},\tilde{y},\tilde{t}),\tilde{t},\tilde{v},\tilde{\gamma}$ and $\tilde{s}$ which are respectively, the perturbed velocity potential in water, perturbed velocity potential in air, surface elevation of the water-air interface, space coordinates, time, air flow velocity, the ratio of the densities of air to water and surface tension.

These dimensionless quantities are related to the corresponding dimensional quantities by the following relations

$$\left.\begin{aligned}&\tilde{\phi}=\sqrt{k_0^3/g}\;\phi,\quad \tilde{\phi}'=\sqrt{k_0^3/g}\;\phi',\quad (\tilde{x},\tilde{y},\tilde{z})=(k_0x,k_0y,k_0z),\\ &\tilde{\varsigma}=k_0\varsigma,\tilde{t}=\omega t,\tilde{v}=\sqrt{k_0/g}\;v,\quad \tilde{\gamma}=\frac{\rho'}{\rho},\;\tilde{s}=Tk^2{}_0/g,\end{aligned}\right\}$$

Where k_0 is some characteristic wave number, g is the acceleration due to gravity, ρ and ρ' are the densities of water and air respectively and T is the dimension surface tension.

In the future, all the quantities will be written in their dimensionless form with their over (~) dropped.

The perturbed velocity potentials ϕ and ϕ' satisfy the following Laplace equations

$$\nabla^2\phi=0 \quad \text{in} \quad -\infty<z<\varsigma \tag{2}$$

$$\nabla^2\phi'=0 \quad \text{in} \quad \varsigma<z<\infty \tag{3}$$

The kinematic boundary conditions to be satisfied at the interface are the following

$$\frac{\partial\phi}{\partial z}-\frac{\partial\varsigma}{\partial t}=\frac{\partial\phi}{\partial x}\frac{\partial\varsigma}{\partial x}+\frac{\partial\phi}{\partial y}\frac{\partial\varsigma}{\partial y} \quad \text{When} \quad z=\varsigma \tag{4}$$

$$\frac{\partial\phi'}{\partial z}-\frac{\partial\varsigma}{\partial t}-v\frac{\partial\varsigma}{\partial x}=\frac{\partial\phi'}{\partial x}\frac{\partial\varsigma}{\partial x}+\frac{\partial\phi'}{\partial y}\frac{\partial\varsigma}{\partial y}, \quad \text{When} \quad z=\varsigma \tag{5}$$

The condition of continuity of pressure at the interface gives

$$\frac{\partial\phi}{\partial t}-\gamma\frac{\partial\phi'}{\partial t}+(1-\gamma)\varsigma-\gamma v\frac{\partial\phi'}{\partial x}=\frac{1}{2}\left\{\left(\frac{\partial\phi}{\partial x}\right)^2+\left(\frac{\partial\phi}{\partial y}\right)^2+\left(\frac{\partial\phi}{\partial z}\right)^2\right\}$$

$$+\frac{\gamma}{2}\left\{\left(\frac{\partial\phi'}{\partial x}\right)^2+\left(\frac{\partial\phi'}{\partial y}\right)^2+\left(\frac{\partial\phi'}{\partial z}\right)^2\right\}+s\left\{1+\left(\frac{\partial\varsigma}{\partial x}\right)^2+\left(\frac{\partial\varsigma}{\partial y}\right)^2\right\}^{\frac{-3}{2}}$$

$$\times\left\{\left(\frac{\partial\varsigma}{\partial x}\right)^2\frac{\partial^2\varsigma}{\partial y^2}+\left(\frac{\partial\varsigma}{\partial y}\right)^2\frac{\partial^2\varsigma}{\partial x^2}-2\frac{\partial\varsigma}{\partial x}\frac{\partial\varsigma}{\partial y}\frac{\partial^2\varsigma}{\partial x\partial y}+\frac{\partial^2\varsigma}{\partial x^2}+\frac{\partial^2\varsigma}{\partial y^2}\right\},$$

When $z = \varsigma$ (6)

Also ϕ and ϕ' should satisfy the following conditions at infinity

$$\frac{\partial \phi}{\partial z} \to 0 \qquad \text{When} \qquad z \to -\infty \tag{7}$$

$$\frac{\partial \phi'}{\partial z} \to 0 \qquad \text{When} \qquad z \to \infty \tag{8}$$

We look for solutions of the above equations (2) and (8)in the following form

$$P = P_{00} + \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}[P_{mn} \exp i(m\psi_1 + n\psi_2) + P^*_{mn} \exp - i(m\psi_1 + n\psi_2)] \tag{9}$$

Where $\psi_1 = k_1 x - \omega\, t$, $\psi_1 = k_1 x - \omega\, t$ and G stands for $\phi, \phi'\, \eta$. In the above summation on the right hand side of equation (9), $(m,n) \neq (0,0)$. The Fourier coefficients $\phi_{00,}\ \phi'_{00,}\ \phi_{mn,}\ \phi'_{mn,}\ \phi^*{}_{mn}\ \phi'^*_{mn,}$ are functions of z, $x_1 = \varepsilon x, t_1 = \varepsilon\, t$ and $\varsigma_{00,}\ \varsigma_{mn,}\ \varsigma^*{}_{mn,}$ are functions of $x_1, y_1\, t_1$. ε is a small ordering parameter measuring the weakness of wave steepness , which is the product of wave amplitude and wave number and * denotes complex conjugate.

The linear dispersion relation for gravity waves

$$(1+\gamma)\omega^2 - 2\gamma\,\omega\, v + \gamma v^2 - (1-\gamma) - s = 0 \tag{10}$$

Which gives the following two values of $\omega_{\pm}$

$$\omega_{\pm} = \left[\gamma v \pm \sqrt{1-\gamma^2 - \gamma v^2 + s(1+\gamma)}\right] / (1+\gamma) \tag{11}$$

Which corresponds to two modes and we designate this two modes as positive and negative modes. The positive mode moves in the positive direction of the x- axis with a frequency $\left[\gamma v + \sqrt{1-\gamma^2 - \gamma v^2 + s(1+\gamma)}\right] / (1+\gamma)$, while the negative mode moves in the negative direction of the x -axis with a frequency $\left[\sqrt{1-\gamma^2 - \gamma v^2 + s(1+\gamma)} - \gamma v\right] / (1+\gamma)$. If v is replace by $-v$ the frequency of the positive mode becomes equal to the frequency of the negative mode. So the results for the negative mode can be obtained from those for the positive mode by replacing v by $-v$. Therefore we have made a nonlinear analysis for the positive mode, and then we have obtained the results for the negative mode by replacing v by $-v$.

For linear stability v should satisfy the following condition

$$|v| < \sqrt{\left[1-\gamma^2 + s(1+\gamma)\right] / \gamma} \qquad 12)$$

So our present analysis will remain valid as long as the dimensionless flow velocity of the air becomes less than the critical velocity $\sqrt{\left[1-\gamma^2 + s(1+\gamma)\right] / \gamma}$. For air flowing over water γ = 0.00129 and this critical value becomes 28.87, for s = 0.075.

III. DERIVATION OF EVOLUTION EQUATIONS

Substituting expansions (9) in equations (2),(3),(7),(8) and then equating the coefficients of $\exp i(m\psi_1 + n\psi_2)$ for (m,n)=[(1,0),(0,1),(2,0),(0,2),(1,1),(-1,1)]

We obtain the following equations:

$$\left(\frac{\partial^2}{\partial z^2} - \Delta^2{}_{mn}\right)\phi_{mn} = 0 \tag{13}$$

$$\left(\frac{\partial^2}{\partial z^2} - \Delta^2{}_{mn}\right)\phi'_{mn} = 0 \tag{14}$$

$$\frac{\partial \phi_{mn}}{\partial z} \to 0 \tag{15}$$

$$\frac{\partial \phi'_{mn}}{\partial z} \to 0 \tag{16}$$

Where Δ_{mn} is the operator given by

$$\Delta^2{}_{mn} = \left\{(m+n) - i\varepsilon\frac{\partial}{\partial x_1}\right\}^2 - \varepsilon^2\frac{\partial^2}{\partial y_1{}^2} \tag{17}$$

The solutions of equations (13) and (14) satisfying boundary conditions (15) and(16) respectively are given by

$$\phi_{mn} = \exp(\Delta_{mn} z) A_{mn} \tag{18}$$

$$\phi'_{mn} = \exp(-\Delta_{mn} z) A'_{mn} \tag{19}$$

In which A_{mn}, A'_{mn} are functions of x_1, y_1 and t_1. For the sake of convenience we take the Fourier transformation of equations (2),(3),(7) and (8) for (m,n)=(0,0). The solutions of these transformed equations becomes

$$\bar{\phi}_{00} = \exp(|\bar{k}|z)\bar{A}_{00} \tag{20}$$

$$\bar{\phi}'_{00} = \exp(|\bar{k}|z)\bar{A}'_{00} \tag{21}$$

Where $\bar{\phi}_{00}$ and $\bar{\phi}'_{00}$ are Fourier transforms of ϕ_{00} and ϕ'_{00} respectively, defined by

$$(\bar{\phi}_{00}, \bar{\phi}'_{00}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(\bar{\phi}_{00}, \bar{\phi}'_{00}) \exp i(\bar{k}_x x_1 + \bar{k}_y y_1 - \bar{\omega} t_1) dx_1 dy_1 dt_1 \tag{22}$$

Where $\bar{k}^2 = (\bar{k}_x{}^2 + \bar{k}_y{}^2)$, $\bar{A}_{00}$ and $\bar{A}'_{00}$ are functions of $\bar{k}_x$ and $\bar{k}_y$ and $\bar{\omega}$.

Again substituting expansions (9) in the Taylor expanded forms of equations (4)-(6) about z=0 and then equating the coefficients of $\exp i(m\psi_1 + n\psi_2)$ for (m,n)=[(1,0),(0,1),(2,0),(0,2),(1,1),(-1,1),(0,0)] on both sides , we get the following equations

$$\left(\frac{\partial \phi_{mn}}{\partial z}\right)_{z=0} + i\left\{(m-n)\omega + i\varepsilon\frac{\partial}{\partial t_1}\right\}\varsigma_{mn} = a_{mn} \tag{23}$$

$$\left(\frac{\partial \phi_{mn}}{\partial z}\right)_{z=0} + i\left\{(m-n)\omega + i\varepsilon\frac{\partial}{\partial t_1}\right\}\varsigma_{mn} - i\nu\left\{(m+n)\omega - i\varepsilon\frac{\partial}{\partial x_1}\right\}\varsigma_{mn} = b_{mn} \tag{24}$$

$$-i\left\{(m-n)\omega + i\varepsilon\frac{\partial}{\partial t_1}\right\}(\phi_{mn})_{z=0} + i\gamma\left\{(m-n)\omega + i\varepsilon\frac{\partial}{\partial t_1}\right\}(\phi'_{mn})_{z=0} + (1-\gamma)\varsigma_{mn}$$

$$+ s\Delta_{mn}\eta_{mn} - i\gamma\nu\left\{(m+n)\omega - i\varepsilon\frac{\partial}{\partial x_1}\right\}(\phi'_{mn})_{z=0} = c_{mn} \tag{25}$$

Where $(\ )_{z=0}$ implies the value of the quantity inside brackets at z = 0 and a_{mn}, b_{mn}, c_{mn} are contributions from nonlinear terms. Now for the above seven values of (m,n), we obtain seven sets of equations,in which we substitute the solutions for ϕ_{mn}, ϕ'_{mn} given by (18)-(21). We now considering the following perturbation expansions for the solutions of above three sets of equations

$$F_{mn} = \sum_{i=1}^{\infty} \varepsilon^i F^i_{mn} \text{ for (m,n)=[(1,0),(0,1)] ; } F_{mn} = \sum_{i=1}^{\infty} \varepsilon^i F^i_{mn} \text{ for (m,n)=[(2,0),(0,2),(1,1),(-1,1),(0,0)]} \tag{26}$$

Where F_{mn} stands for A_{mn} , A'_{mn} and ς_{mn} .

Substituting expansions (26) in the above three sets of equations and then equating coefficients of various powers of ε on both sides,we obtain a sequence of equations. From the first order (i.e lowest order) and second order equations corresponding to (23) and (24) of the first set of equations we obtain solutions for A_{mn} , A'_{mn} and ς_{mn} ; (m,n)=)=[(1,0),(0,1), (2,0),(0,2),(1,1),(-1,1),(0,0)].We now take the following transformations, following Pierce and Knobloch [14]of all perturbed quantities in slow space coordinates and time given by

$$\xi_+ = x_1 - c_g t_1, \quad \xi_- = x_1 + c_g t_1, \quad \varsigma = y_1, \quad \tau = \varepsilon^2 t_1 \quad \text{Where } c_g = \left(\frac{dw}{dk}\right)_{k=1} \text{ is the group velocity.}$$

Now arranging different terms of we obtain the third order nonlinear evolution equation for ς_{10} :

$$\frac{\partial \varsigma^{(1)}_{10}}{\partial \tau_1} + \delta_1 \frac{\partial^2 \varsigma^{(1)}_{10}}{\partial \xi_+^2} + \delta_2 \frac{\partial^2 \varsigma^{(1)}_{10}}{\partial \eta^2} = \gamma_1 |\varsigma^{(1)}_{10}|^2 \varsigma^{(1)}_{10} + \gamma_2 |\varsigma^{(1)2}_{10}|\varsigma^{(1)}_{10} \tag{27}$$

Again we get the third order nonlinear evolution equation for ς_{01} :

$$-\frac{\partial \varsigma^{(1)}_{01}}{\partial \tau_1} + \delta_1 \frac{\partial^2 \varsigma^{(2)}_{01}}{\partial \xi_-^2} + \delta_2 \frac{\partial^2 \varsigma^{(1)}_{01}}{\partial \eta^2} = \gamma_1 |\varsigma^{(1)}_{01}|^2 \varsigma^{(1)}_{01} + \gamma_2 |\varsigma^{(1)2}_{01}|\varsigma^{(1)}_{01} \tag{28}$$

If we put $\nu = 0, \gamma = 0$ in equation (27) and (28) then we get nonlocal mean field evolution equations in the third order for infinite depth water. These reduce equations becomes the same as equations (2) of Janssen [6]

IV. STABILITY ANALYSIS

The uniform wave train solutions of equations (27) and (28) are given by

$$\varsigma_{10} = \varsigma_{10}^{(0)} = \alpha_0 \exp(i\Delta\omega\tau), \quad \varsigma_{01} = \varsigma_{01}^{(0)} = \alpha_0 \exp(i\Delta\omega\tau), \tag{29}$$

Where α_0 is real constant and the nonlinear frequency shift $\Delta\omega$ is given by

$$\Delta\omega = -(\delta_1 + \delta_2)\alpha_0^2 \tag{30}$$

Finally we obtain the following nonlinear dispersion relation

$$\Omega = \left\{\gamma_1\lambda^2\left(\gamma_1\lambda^2 + 2\delta_1\alpha_0{}^2\right)\right\}^{\frac{1}{2}} \tag{31}$$

From the relation (31) we observe that instability occurs when $\gamma_1\delta_1 < 0$ for long wave length that is for $\lambda \to 0^+$. when instability condition is fulfilled, the growth rate of instability

$$\Gamma = [-\gamma_1\lambda^2(\gamma_1\lambda^2 + 2\delta_1\alpha_0{}^2)]^{\frac{1}{2}} \tag{32}$$

For $\lambda^2 = -\delta_1\alpha_0{}^2 / \gamma_1$, the maximum growth rate of instability

$$\Gamma_m = |\delta_1|\alpha_0{}^2 \tag{33}$$

At marginal stability

$$\gamma_1\lambda^2 + 2\delta_2\alpha_0{}^2 = 0$$

And the wave number

$$\lambda = \frac{\sqrt{2}\delta_1\alpha_0}{\sqrt{|\gamma_1\delta_1|}} \tag{34}$$

In Figures 1 and 2 the maximum growth rate Γm of instability which can be obtained from equation (33) has been plotted against wave steepness α_0 for some different values of dimensionless wind velocity v and for s=0.075. From these graphs it is found that for waves with sufficiently small waves numbers the maximum growth rate of instability Γm increases steadily with the increase of wave steepness α_0 . The maximum growth rate also increases with the increase of dimensionless wind velocity v . The growth rate is found to be appreciably much higher for dimensionless wind velocity approaching its critical value.

Again in Figures 3 and 4 the wave number λ at marginal stability which can be obtained from equation (34) has been plotted against wave steepness α_0 for some different values of dimensionless wind velocity *v*. From these graphs it is observed that the instability regions are shortened with the increase of the absolute value of the wind velocity.

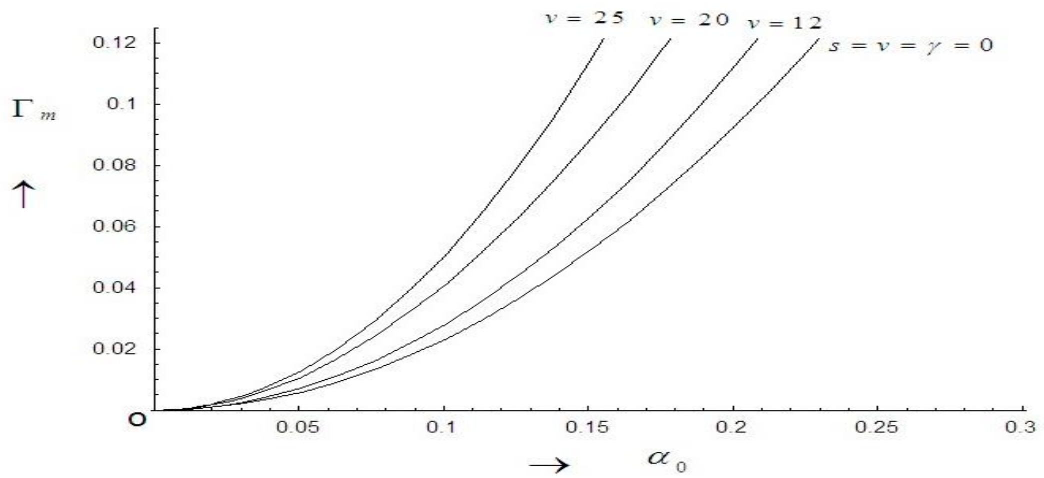

Figure 1: Maximum growth rate of instability Γ_m against wave steepness α_0 for some different values of dimensionless wind velocity *v*. Here γ = 0.00129 and *s* = 0.075 for all the graphs except for the one with *s* = *v* = *γ* = 0 written on the graph.

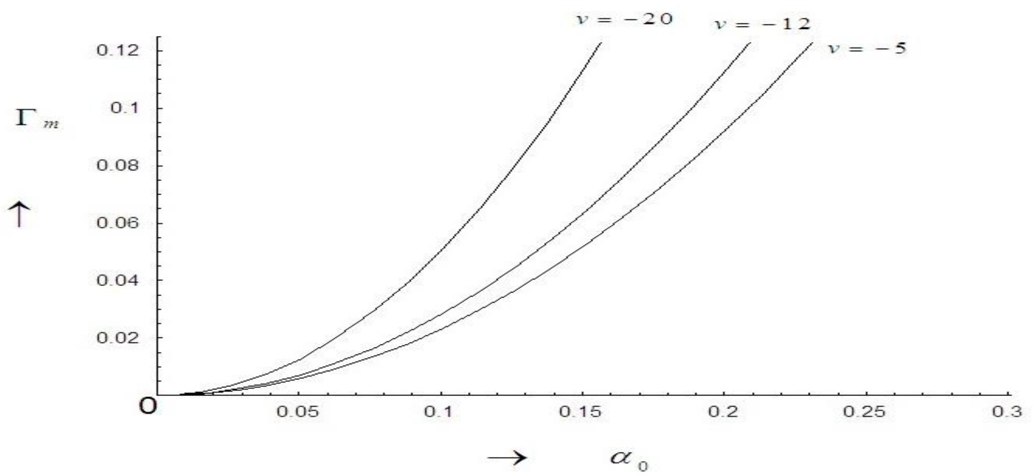

Figure 2: Maximum growth rate of instability Γ_m against wave steepness α_0 for some different values of dimensionless wind velocity *v*. Here γ = 0.00129. And *s* = 0.075 for all the graphs

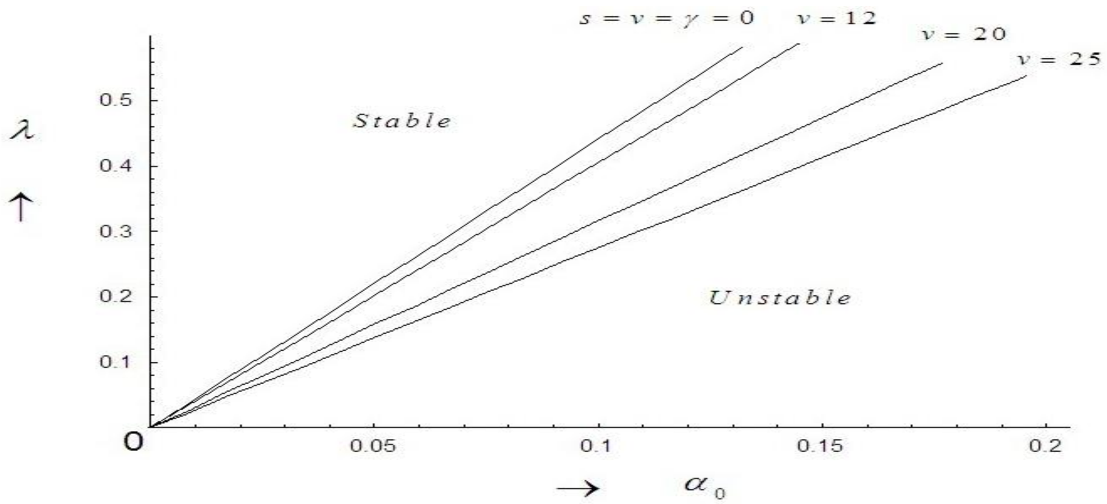

Figure 3: Wave number λ at marginal stability against wave steepness α_0 for some different values of dimensionless wind velocity *v*. Here γ = 0.00129 and *s* = 0.075 for all the graphs except for the one with *s* = *v* = γ = 0 written on the graph.

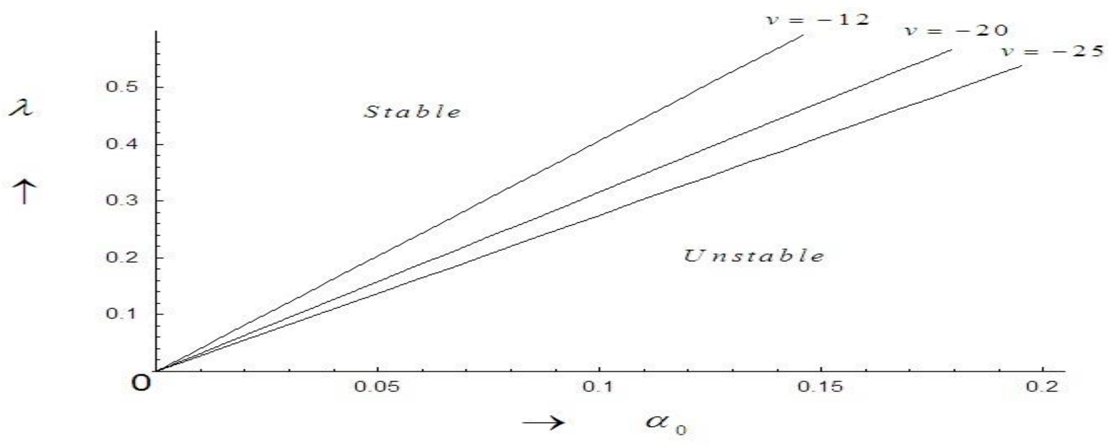

Figure 4: Wave number λ at marginal stability against wave steepness α_0 for some different values of dimensionless wind velocity *v*. Here γ = 0.00129 and *s* = 0.075 for all the graphs.

V. DISCUSSION AND CONCLUSION

The third order nonlinear evolution equations have been derived by Pierce and Knobloch [9] for two counterpropagating capillary gravity wave packets on the surface of water of finite depth. The resulting equations are asymptotically exact and nonlocal and generalize the equations derived by Djordjevic and Redekopp [5] for counterpropagating waves. Our paper is an extension of the evolution equations derived by Pierce and knobloch [9] for an infinite depth water and in the presence of wind flowing over it. From these evolution equations instability condition is obtained and graphs are plotted showing maximum growth rate of instability Γm against wave steepness α_0 for some different values of dimensionless wind velocity *v*. From the graphs it is found that the maximum growth rate of instability Γm increases steadily with the increase of wave steepness α_0 .

The maximum growth rate also increases with the increase of dimensionless wind velocity *v*. The growth rate of instability is found to be appreciably much higher for dimensionless wind velocity approaching its critical value. Graphs are also plotted for the wave number λ at marginal stability against wave steepness α_0 for some different values of dimensionless wind velocity *v*. From the graphs it is observed that the instability regions are shortened with the increase of the absolute value of the wind velocity.

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APPENDIX – A

Coefficients of the Evolution Equations (27) and (28):

$$\gamma_0 = \left\{2\gamma v\omega - 2\gamma v^2 + (1-\gamma) + 3s\right\} / \left\{(1+\gamma)\omega^2 - \gamma v\omega\right\},$$

$$\gamma_1 = \left\{2\gamma v c_g - \gamma v^2 + (1+\gamma)c_g^2 + 3s\right\} / \left\{2(1+\gamma)\omega^2 - 2\gamma v\omega\right\},$$

$$\gamma_2 = \left\{(1-\gamma)c_g^2 - 2\gamma v c_g + 3s\right\} / \left\{4(1+\gamma)\omega^2 - 4\gamma v\omega\right\},$$

$$\delta_1 = [(2\omega^4 + 6\omega^2 - 9s) + \gamma\{\frac{21}{2}(\omega^2 + v^2) + 2(2 + p_1)(\omega - v)(\omega - v - 2\omega^2) - (1+2p_1)\omega + 15\omega v\} + \gamma v(\omega - v)(6p_1 + 9)] / [12\omega^2 - 8\omega^4 - \gamma(\omega - v)^2],$$

$$\delta_2 = [31\omega^4 - 23\omega^2 + s^2(1-\gamma) - 8s + 8\gamma(\omega - v)^2 - \gamma v(8 - \omega p_2)] / [8\omega^4 - 6\omega^2 - 2\gamma(\omega - v)^2],$$

$$c_g = [2\gamma v\omega - 2\gamma v^2 + (1-\gamma) + 3s] / [2(1+\gamma)\omega - 2\gamma v],$$

$$p_1 = [\omega^2 - 3\gamma(\omega - v)^2] / [2\omega^2(\gamma - 1) - 2\gamma\omega(v + \omega) + 3(1-\gamma)\}],$$

$$p_2 = [2\omega^2(1+\gamma) - 4\gamma v^2] / [2\omega^2(\gamma + 1) + 2\gamma\omega(\omega - v) - 3(1-\gamma)\}],$$

Nomenclature:

$\left.\begin{array}{l}\gamma_i (i = 0,1,2) \\ \delta_i (i = 1,2)\end{array}\right\}$ - coefficients given in the Appendix,

ε - Slowness parameter, α - wave steepness, ς - elevation of the air water interface, ω - frequency, γ - Ratio of densities of air to water, $\Delta\omega$ - frequency shift, Ω - perturbed frequency at marginal stability. g – Acceleration due to gravity, λ - wave number, s – dimensionless surface tension, t – time, v - air flow velocity, Γ_m - growth rate of instability .